Many beta points too far: is 42 really the answer?

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A good decision is based on knowledge and not on numbers.
Plato

Thanks to Prof. S. Schäffler (UniBw Munich/Neubiberg) for explaining to the ignorant author some concepts of global optimization
(but he is not responsible for anything said here)

E R A

Terminus and Mike Box

The god of boundaries and limits

All should know their limits

Terminus and Mike Box

The god of boundaries and limits

All should know their limits

There is no strength in numbers,
have no such misconception.
(Uriah Heep, Lady in Black)

I. Structuralism

(What I think IMHO should be done)

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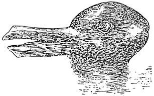
Gestalt Switches

Thomas S. Kuhn

Kuhn argued in *The structure of scientific revolutions* Kuhn (1996) that these are caused by *gestalt switches*. One looks at the known fact or structure from different angle or perspective and suddenly one sees something different. But also in the time between revolutions science progresses by many small *gestalt switches* (see Kuhn (1996), p. 181 and Kuhn (1970), p. 249, note 3). Also in structural reliability there was a sequence of such switches.



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Structuralism

Jean Piaget

Structuralism is a scientific methodology emphasizing the relations between the elements of the subject as main topic of the study, for a description see Piaget (1971). After Rickart (1995) "structuralism" can be defined as a *method of analyzing a body of information with respect to its inherent structure*.

Mathematical structuralists think that mathematics is fundamentally concerned with structures, or with the relations mathematical objects bear to each other in virtue of belonging to some structure.

Charles E. Rickart

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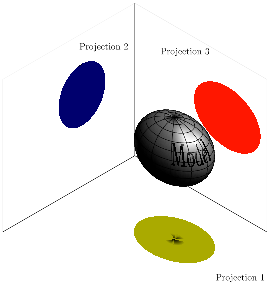
A Gestalt Switch towards Structuralism

Structural reliability should make a gestalt switch towards a structuralist view of reliability problems. This becomes more and more necessary, since the problem structures are getting more complex.
Try to identify the relevant substructure as primary target, failure probabilities then as secondary target.

The changing shapes

Von der Vergangenheit trennt uns nicht ein Abgrund, sondern die veränderten Verhältnisse. (A.Kluge)



(a) The reliability problem at the beginning

The changing shapes

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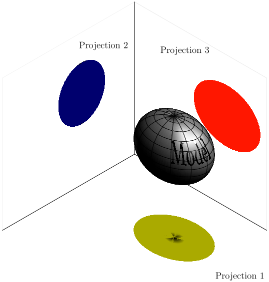
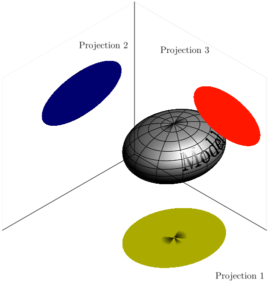


(a) The reliability problem at the beginning



(b) The reliability problem evolving

The changing shapes

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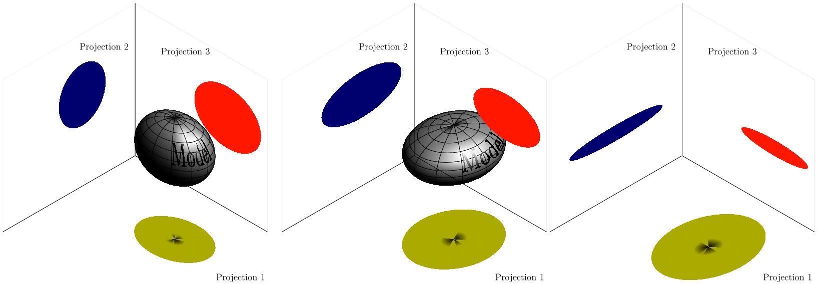
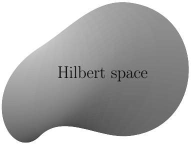


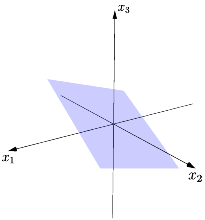
(a) The reliability problem at the beginning (b) The reliability problem evolving (c) The reliability problem now

Figure: The varying shapes of the reliability problem

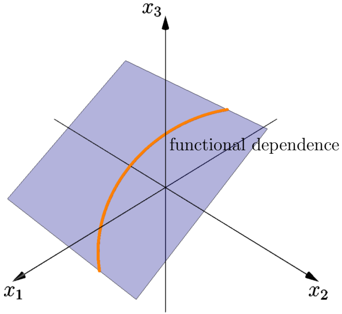
Development of structural model



Underlying Hilbert space



Dimension reduction



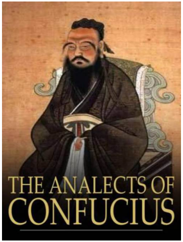
Building a functional model

II. Subset Simulation

(Which IMHO is wrong)

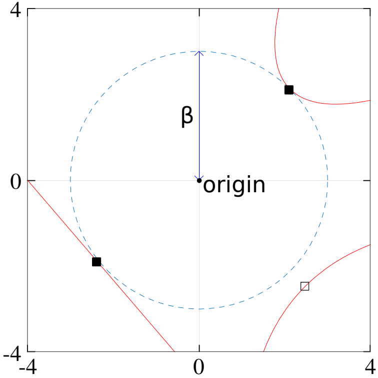
Confucius on Names

One day, a disciple asked Confucius: "If a king were to entrust you with a territory which you could govern according to your ideas, what would you do first?"
Confucius replied: "My first task would certainly be to rectify the names."
The puzzled disciple asked: "Rectify the names? Is this a joke?"
Confucius replied: "If the names are not correct, if they do not match realities, language has no object. If language is without an object, action becomes impossible..."
(The Analects of Confucius, Book 13, Verse 3)



The Basic Problem

In standard normal space with pdf $f(\mathbf{u}) = (2\pi)^{-n/2} \exp(-|\mathbf{u}|^2/2)$ approximate $P(F) = \mathrm{P}(\{g(\mathbf{u}) < 0\})$. This is the *REAL THING*, nothing else, and also SuS is an approach to solve this.



In the standard normal space the design points (filled black squares) have to be found. Then with FORM/SORM asymptotic approximations are derived:

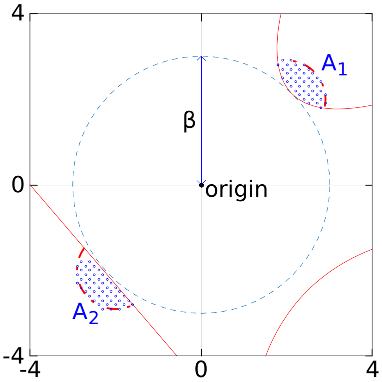
$$\mathrm{P}(F) \sim \Phi(-\beta) \cdot C, \ \beta \to \infty$$

F(S)ORM First (Second) Order Reliability Methods referring to the order of the Taylor expansion.

ERA

The Basic Problem in SuS Formulation

In standard normal space with pdf $f(\mathbf{u}) = (2\pi)^{-n/2} \exp(-|\mathbf{u}|^2/2)$:



Doing asymptotic analysis without calculus. In the standard normal space the design areas A_1 and A_2 (neighborhoods of the design points) have to be found and their probability content estimated for an asymptotic approximation.

$$\mathrm{P}(F) \sim \mathrm{P}(A_1) + \mathrm{P}(A_2), \ \beta \to \infty$$

This is a result derived by M. Hohenbichler (see Breitung (1994), p. 53).

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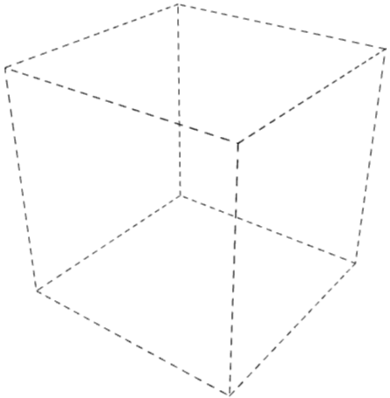
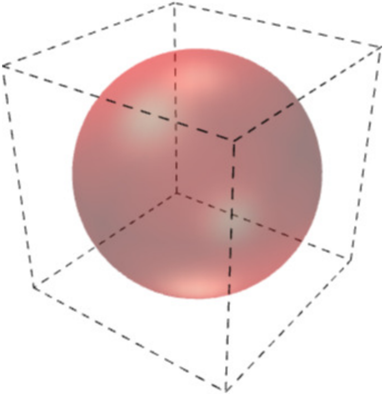
Mathematical and engineering logic I

The cube denotes a set of problems. Assume a mathematician finds a solution idea. He will derive a theorem valid in the red sphere.



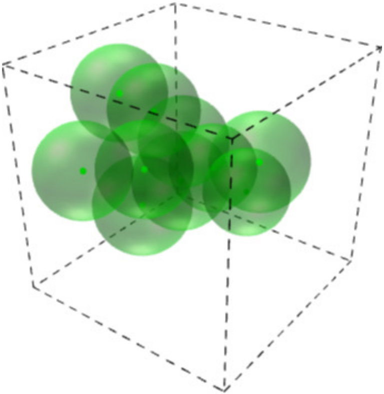
Mathematical solution I

The cube denotes a set of problems. Assume a mathematician finds a solution. He will derive a theorem valid in the red sphere.
An engineer will check his heuristics



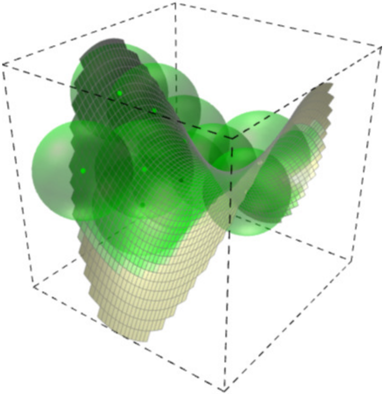
Engineering solution I

An engineer will check his solution idea by calculating a number of examples (green dots). So he will get an idea that the method works for similar cases (green spheres).



Hidden assumption I

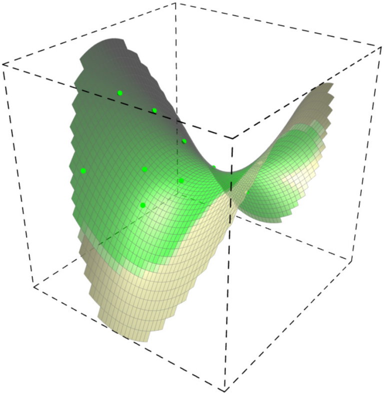
But since in the calculation of these examples it is not clearly specified what properties these examples have, it might happen that there is a hidden assumption common to all examples (grey surface).



▶

¡restricted validity I

So in fact taking into account this hidden assumption, the method is valid only for the cases where this assumption is fulfilled.
Green surface part.



Credo of Subset Simulation (SuS)

Zuev et al. (2012):

Subset Simulation provides an efficient stochastic simulation algorithm for computing failure probabilities for general reliability problems without using any specific information about the dynamic system other than an input-output model. This independence of a systems inherent properties makes Subset Simulation potentially useful for applications in different areas of science and engineering where the notion of "failure" has its own specific meaning,...

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Monahan (2011) p. 394:

. . . For MCMC, an extremely naive user can generate a lot of output without even understanding the problem. The lack of discipline of learning about the problem that other methods require can lead to unfounded optimism and confidence in the results.

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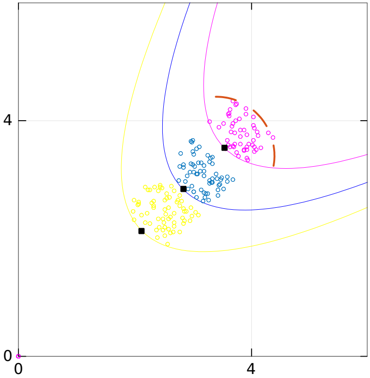
Monahan (2011) p. 394:

...For MCMC, an extremely naive user can generate a lot of output without even understanding the problem. The lack of discipline of learning about the problem that other methods require can lead to unfounded optimism and confidence in the results.

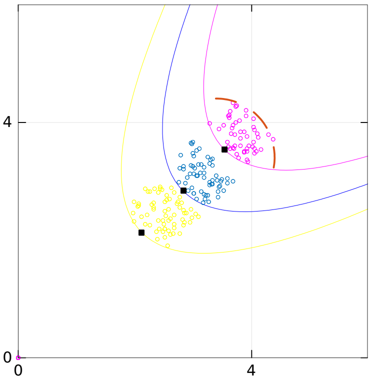
Promise of SuS: You can do everything without understanding anything!

The Standard SuS Example



From a larger value $c_1 > 0$ the failure regions $F_j = \{g(\mathbf{u}) < c_j\}$ with $c_1 > c_2 > ..c_n = 0$ are made successively smaller until the original failure domain $\{g(\mathbf{u}) < 0\}$ is reached. Here also the design points for the domains F_j are shown. Using Hohenbichler's lemma now estimate the probability from the points in magenta.

Iteration: Design Points and Regions



In the SS approach the relevant areas of F_n are found near the last region in F_{n-1} [a]. In SORM this corresponds to searching the next design point for F_j in the neighborhood of the last for F_{j-1}. *Sounds reasonable?*

A really grave problem in mathematics is that not everything which sounds reasonable is correct.

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[a] *"Given that we have found a failure point $\theta \in F_{n-1}$, it is reasonable to expect that more failure points are located nearby"*

This does not work as advertised!

Some Warnings Ignored

Rüdiger Rackwitz

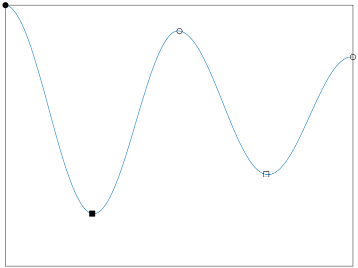
As Rackwitz (2001) said, an important step in the development of methods is to show where they do not work, i.e. to find the limits of the applicability of the concept and to construct counterexamples.

And Hooker (1995) said that the most important point is to understand an algorithm not to make it efficient. `http://repository.cmu.edu/tepper/197/`

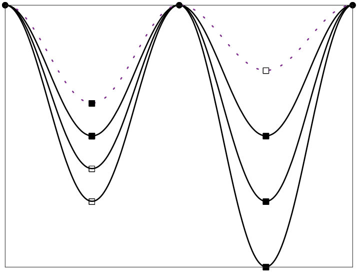
John N. Hooker

Sequential determination of global extrema

Global and local extrema of functions: minima are shown by squares, maxima by circles, filled symbols are global extrema



(a) Local and global extrema of a function



(b) The global minima of a function depending on a parameter

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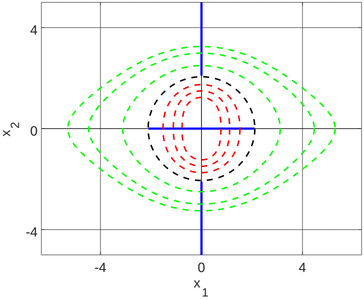
A Simple Example with Smooth Functions

The position of global minima under constraints. Given a LSF:

$$g(u_1, u_2) = \beta - u_1^2 - \frac{|\mathbf{u}|^2}{b^2} u_2^2 = \beta - u_1^2 - \frac{(u_1^2 + u_2^2)u_2^2}{b^2} = \beta - u_1^2 - \frac{u_2^1 + u_2^4}{b^2}$$

The points with global minimum distance to the origin under $g(u_1, u_2) = c$ lie always on the axes (on the blue line segments).

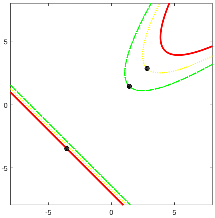


The minimum distance points jump when the black circle is reached.

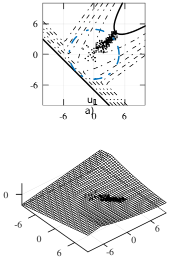
This is a normal behavior in global optimization!

Series System with SuS

Find the point with minimal distance to the origin — design point — on the domain bounded by the thick red curve $\{g(\mathbf{u}) = 0\}$.



(a) The contours for g



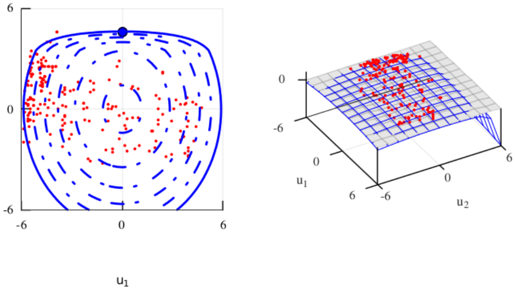
(b) SuS algorithm for g

$$g_1(u_1, u_2) = 0.6 + \frac{(u_1 - u_2)^2}{40} - \frac{u_1 + u_2}{10\sqrt{2}}, \qquad g_2(u_1, u_2) = 5 + \frac{u_1 + u_2}{\sqrt{2}}$$

$$g(u_1, u_2) = \min(g_1, g_2)$$

Remember Wild Bill Hickok. You also have to look behind you!

Extrapolation with SuS



LSF: $g(x_1, x_2) = 0.1 \cdot (52 - 1.5 \cdot x_1^2 - x_2^2)$

$$F(x_1) = \Phi(x_1),\ \ F_2(x_2) = \begin{cases} \Phi(x_2) & ,\ x_2 \leq 3.5 \\ 1 - {x_2}^c & ,\ x_2 > 3.5 \end{cases}$$

Global minimization and SuS

It is not possible to find the design point (global minimum point on $g(\mathbf{u}) = 0$) by a sequential method for $c_1 > c_2 > \ldots c_n = 0$

$$|\mathbf{u}^j| = \min_{g(\mathbf{u}) \leq c_j} |\mathbf{u}|, \quad \mathbf{u}^j \to \mathbf{u}^{j+1} \to \mathbf{u}^n = \min_{g(\mathbf{u}) \leq 0} |\mathbf{u}|$$

This works in examples with a *Simple Simon* geometry, but not in general. If someone says, SuS is not an attempt to global minimization, **what is it then**?
And if someone says, SuS does not work for such simple examples, remember: *Hic Rhodus, hic salta!*
The main problem in global optimization is to avoid local extrema and to get out of them if stuck there. Unfortunately this is complicated, it is not enough to run some MCMC's and wait.

III. Onion Concept

(Which IMHO might help a little)

In global minimization for structural reliability one has to find the global minimum point $\mathbf{u}^*$:

$$|\mathbf{u}^*| = \min_{g(\mathbf{u})\leq 0} |\mathbf{u}|$$

Define the spheres $S(y) = \{\mathbf{u}; \mathbf{u}| = y\}$, this can be done finding the beta sphere defined by

$$\beta = \min_{y>0}\{S(y); \min_{\mathbf{u}\in S} g(\mathbf{u}) \leq 0\}$$

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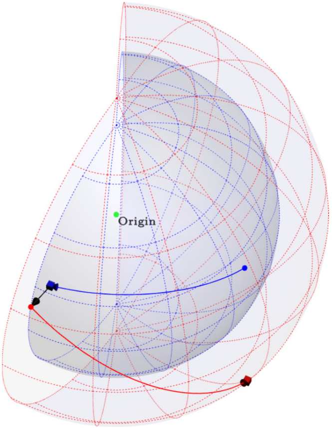
The Onion Concept



In the original FORM/SORM concept the design point is searched by solving the Lagrangian system:

$$\begin{aligned} \mathbf{u} + \lambda \nabla g(\mathbf{u}) &= \mathbf{0} \\ g(\mathbf{u}) &= 0 \end{aligned}$$

Now, instead one searches the extrema of the LSF on a centered sphere with radius γ in an iterative way

$$\begin{aligned} \nabla g(\mathbf{u}) + \mu \mathbf{u} &= \mathbf{0} \\ |\mathbf{u}|^2 - \gamma^2 &= 0 \end{aligned}$$

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Onion Method Example

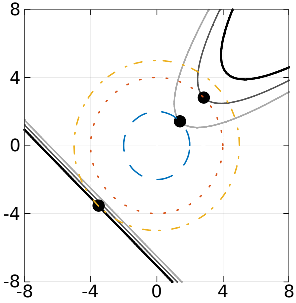


Figure: The contours for g

Table: Iteration steps

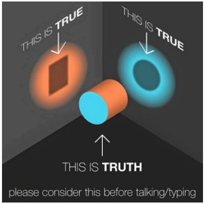
Step	Iteration Point
1	$(1, 2)$
2	$(-1.58, -1.58)$
3	$(-4.58, -4.58)$
4	$(-3.10, -3.10)$
5	$(-3.71, -3.71)$
6	$(-3.46, -3.46)$
7	$(-3.57, -3.57)$
8	$(-3.52, -3.52)$
9	$(-3.54, -3.54)$
10	$(-3.53, -3.53)$

IV. Philosophy of science

Against Method

Paul Feyerabend

This is not an appeal to go forward in a specific direction but to see things from a broader perspective and to try out various methods and concepts. Since science is — as Feyerabend (1993) says — in principle an anarchistic enterprise. And to give a further quote from him, all methodologies have their limits even the most obvious ones. So there is plenty of room for new research.



Instead of Conclusions an Advice from Star Trek

Episode *Phage* from Voyager

Kes: How does a real doctor learn to deal with patients' emotional problems, anyway?
The Doctor: They learn from experience.
Kes: Aren't you capable of learning?
The Doctor: I have the capacity to accumulate and process data, yes.
Kes: **Then I guess you'll just have to learn - like the rest of us.**

Thank you for your attention

Some manuscripts: Researchgate, arxiv, osf

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